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### THESIS

AUV STEERING PARAMETER IDENTIFICATION FOR  
IMPROVED CONTROL DESIGN

by

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June 2001

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**AUV STEERING PARAMETER IDENTIFICATION FOR IMPROVED  
CONTROL DESIGN**

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Submitted in partial fulfillment of the  
requirements for the degree of

**MASTER OF SCIENCE IN MECHANICAL ENGINEERING**


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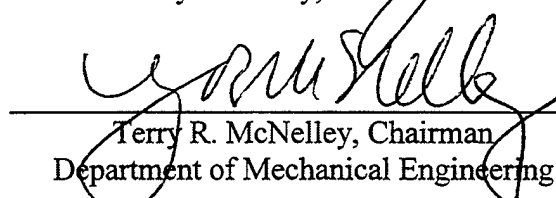
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## **ABSTRACT**

Any effort to provide precision control for an Autonomous Underwater Vehicle requires an accurate estimation of both the vehicles physical and hydrodynamic parameters. Here a vehicle model for controlled steering behaviors was developed and the hydrodynamic parameters were calculated from actual data obtained from operation. The steering equation parameters are based on a least squares fit to sideslip and turn rate data using maximum likelihood of batch processing. In this way, a more accurate simulation has been found for the development of a track controller that stably drives the vehicle between mission waypoints. Prediction accuracy of the model was better than ninety-five percent over the data set used.

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# **I. INTRODUCTION**

## **A. GENERAL BACKGROUND AND LITERATURE**

The increased desire to use Autonomous Underwater Vehicles (AUVs) and Unmanned Undersea Vehicles (UUVs) for commercial and military applications has led to a great deal of research in this field over the last decade. Specifically, the military need for clandestine mine reconnaissance is now being solved using AUVs. A high degree of maneuverability in shallow water conditions means these vehicles must have high performance steering controllers requiring accurate computer models. Although computer models have facilitated the estimation of fluid parameters on vehicles, the building of accurate models to match complex shapes is tedious and time consuming. An exact calculation of the dynamic parameters of an AUV is impossible. Even with an estimation of these parameters, there remains the question of validation of the results obtained. To get the hydrodynamic properties for marine vehicles generally one of the three following methods is normally used:

1. Detailed computer modeling as previously mentioned;
2. A tow tank. In a tow tank, generally a model (models of large vessels are used due to size restrictions) of the vehicle is pulled through the water and maneuvered while maintaining a constant speed. During this entire time, instruments are used to measure the forces acting on the vehicle. This process is very informative and provides very good results. But, it is very time consuming and expensive to accomplish; and
3. By approximating the shape of the vehicle to known shapes and using this approximation to calculate some coefficients and using system identification methods for others.

Apart from the calculation of the hydrodynamic masses, hydrodynamic coefficients,  $Y_v$ ,  $Y_r$ ,  $N_v$ ,  $N_r$ ,  $Y_\delta$  and  $N_\delta$  are not easily computed, and when measured using 2, do not always match field data from 3.

## **B. SCOPE OF THIS WORK**

The overall problem of marine vehicle hydrodynamics is very complex and diverse. This study will focus on the identification of the NPS ARIES AUV steering system hydrodynamic parameters. In particular the values of added mass for sway and yaw are determined from physical measurements, which are adapted to an approximate shape. Normal system identification techniques [Ljung et al.] obtain values for a canonical set of transfer function parameters for which  $Y_v$ ,  $Y_r$ ,  $N_v$  and  $N_r$  (defined in Chapter II) cannot be extracted uniquely. However, with turn rate ( $r$ ), sideslip velocity ( $v$ ), and rudder angle ( $\delta$ ) measurements, it is possible to get a uniquely identifiable estimate of each parameter  $N_v$ ,  $N_r$ ,  $Y_v$  and  $Y_r$ . In this case, we use a set of data obtained from ARIES and use a least mean square batch regression process to obtain unique values.

Previous literature on this subject includes a major work by Abkowitz, (1980) using a tanker. In which the estimated hydrodynamic coefficients were obtained using an extended Kalman Filter. The results were inconclusive however, because they did not match model test data and the ship had to be built in order to get test results.

Chapter II discusses the equations of motion for an underwater vehicle and AUV modeling. It also includes the calculations, formulas and approximations used and the reasoning behind each.

Chapter III discusses the theory of parameter identification from vehicle measurements and the methodology used in this report.

Chapter IV provides information on the data collected by the ARIES AUV and the processing required to obtain the final results.

Chapter V proves that the methodology used in this report provides an accurate way to obtain hydrodynamic coefficients from vehicle data.

Chapter VI lists the conclusions of this report derived from Chapter V.

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## II. EQUATIONS OF MOTION AND AUV MODELING

### A. GENERALIZED STEERING EQUATIONS OF MOTION

This section will discuss the equations of motion for a rigid body in an underwater environment leading to the model used for AUV steering control design and the simplifications made for the analysis of the NPS AUV.

The forces acting on the AUV determine its motion and are independent of the inertial position. The equations of motion are given using Euler equations for the body fixed frame (Greenwood, 1980). The six simplified equations of motion given by (Healey, 1995) are:

#### SURGE EQUATION OF MOTION

$$m[\dot{u}_r - v_r r + w_r q - x_G(q^2 + r^2) + y_G(pq - \dot{r}) + z_G(pr + \dot{q})] + (W - B)\sin \theta = X_f \quad (1)$$

#### SWAY EQUATION OF MOTION

$$m[\dot{v}_r + u_r r - w_r p + x_G(pq + \dot{r}) - y_G(p^2 + r^2) + z_G(qr - \dot{p})] - (W - B)\cos \theta \sin \phi = Y_f \quad (2)$$

#### HEAVE EQUATION OF MOTION

$$m[\dot{w}_r - u_r q + v_r p + x_G(pr - \dot{q}) + y_G(qr + \dot{p}) - z_G(p^2 + q^2)] + (W - B)\cos \theta \cos \phi = Z_f \quad (3)$$

#### ROLL EQUATION OF MOTION

$$I_x \dot{p} + (I_z - I_y)qr + I_{xy}(pr - \dot{q}) - I_{yz}(q^2 - r^2) - I_{xz}(pq + \dot{r}) + m[y_G(\dot{w} - u_r q + v_r p) - z_G(\dot{v}_r + u_r r - w_r p)] - (y_G W - y_B B)\cos \theta \cos \phi + (z_G W - z_B B)\cos \theta \sin \phi = K_f \quad (4)$$



### PITCH EQUATION OF MOTION

$$I_y \dot{q} + (I_z - I_x)pr - I_{xy}(qr + \dot{p}) + I_{yz}(pq - \dot{r}) + I_{xz}(p^2 - r^2) - m[x_G(\dot{w} - u_r q + v_r p) - z_G(\dot{u}_r - v_r r + w_r q)] + (x_G W - x_B B) \cos \theta \cos \phi + (z_G W - z_B B) \sin \theta = M_f \quad (5)$$

### YAW EQUATION OF MOTION

$$I_z \dot{r} + (I_y - I_x)pq - I_{xy}(p^2 - q^2) - I_{yz}(pr + \dot{q}) + I_{xz}(qr - \dot{p}) + m[x_G(\dot{v}_r + u_r r - w_r p) - y_G(\dot{u}_r - v_r r + w_r q)] - (x_G W - x_B B) \cos \theta \sin \phi - (y_G W - y_B B) \sin \theta = N_f \quad (6)$$

The variables  $u$ ,  $v$  and  $w$  are the component velocities expressed in terms of a body fixed coordinate system and the  $r$  subscript denotes that the velocity is with respect to the water. The variables  $p$ ,  $q$  and  $r$  are the components of angular velocity expressed in the body fixed coordinate system.  $W$  and  $B$  are weight and buoyancy and  $I$  is a moment of inertia term. The  $x$ ,  $y$  and  $z$  variables with the  $B$  and  $G$  subscript come from the position difference between the center of gravity, center of buoyancy and the center of the vehicle and the effects of gravity and buoyancy. All of the right hand terms are the forces acting on the AUV acting in the associated direction.

Further simplifications can be made to Equations 1 thru 6 with the following assumptions for the AUV:

1. That the vehicle is neutrally buoyant,  $B = G$ .
2. Horizontal plane motion,  $[w_r, p, q, Z, \phi, \theta, x_G, y_G] = 0$ .
3. The forward speed  $u_r$  is a constant  $U_0$ .

4. Surge motion changes are neglected.

5. Roll coupling is neglected.

These assumptions reduce the steering equations of motion to:

$$u_r = U_o \quad (7)$$

$$m\dot{v}_r = -mU_o r + \Delta Y_f(t) \quad (8)$$

$$I_{zz}\dot{r} = \Delta N_f(t) \quad (9)$$

$$\dot{\psi} = r \quad (10)$$

$$\dot{X} = U_o \cos \psi - v_r \sin \psi + U_{cx} \quad (11)$$

$$\dot{Y} = U_o \sin \psi - v_r \cos \psi + U_{cy} \quad (12)$$

Now that the equations of motion have been reduced, the forces acting on the AUV must be addressed. Recognizing that under steady motion there is a balance between lift, drag, buoyancy, weight, and propulsion provides the starting point to develop the components  $\Delta Y_f(t)$  and  $\Delta N_f(t)$ . The variables  $\Delta Y_f(t)$  and  $\Delta N_f(t)$  represent forces as a function of the vehicles dynamic parameters. Equation 13 assumes that all motion variables to be small compared to vehicle speed and the  $\delta$  implies a 'small change in'.

$$\delta F = f(u_r, v_r, w_r, p, q, r, du_r/dt, dv_r/dt, dw_r/dt, dp/dp, dq/dt, dr/dt, t) \quad (13)$$

Simplifying Equation 13 is based on the following rational that the AUV is generally symmetric about its longitudinal axis and that fluid flow in one direction will

not greatly affect an orthogonal direction. The expression for the transverse force becomes:

$$Y_f = Y_{\dot{v}_r} \dot{v}_r + Y_{v_r} v_r + Y_{\dot{r}} \dot{r} + Y_r r \quad (14)$$

and

$$Y_{\dot{v}_r} = \frac{\partial Y_f}{\partial \dot{v}_r}; \quad Y_{v_r} = \frac{\partial Y_f}{\partial v_r}; \quad Y_{\dot{r}} = \frac{\partial Y_f}{\partial \dot{r}}; \quad Y_r = \frac{\partial Y_f}{\partial r};$$

evaluated at the forward speed  $U_0$ . The coefficient  $Y_{\dot{v}_r}$  is the added mass in sway, evaluated with the vehicle in forward motion,  $Y_{\dot{r}}$  is added mass induced by yaw,  $Y_{v_r}$  is a coefficient of sway force from side slip, and  $Y_r$  is a coefficient of sway force from yaw. The rudder when actuated will produce a lift force that when linearized is expressed as  $Y_\delta \delta_r(t)$ . Likewise the rotational force is developed as:

$$N_f = N_{\dot{v}_r} \dot{v}_r + N_{v_r} v_r + N_{\dot{r}} \dot{r} + N_r r \quad (15)$$

and

$$N_{\dot{v}_r} = \frac{\partial N_f}{\partial \dot{v}_r}; \quad N_{v_r} = \frac{\partial N_f}{\partial v_r}; \quad N_{\dot{r}} = \frac{\partial N_f}{\partial \dot{r}}; \quad N_r = \frac{\partial N_f}{\partial r};$$

evaluated at the forward speed  $U_0$ . The coefficient  $N_{\dot{v}_r}$  is the added mass moment of inertia in sway, evaluated with the vehicle in forward motion,  $N_{\dot{r}}$  is added mass moment induced by yaw,  $N_{v_r}$  is a coefficient of yaw moment from side slipping, and  $N_r$  is a coefficient of sway moment from yaw. The rudder when actuated will produce a force that linearized, is  $N_\delta \delta_r(t)$ . The equations of motion in the horizontal plane are given by:

$$m \dot{v}_r = -m U_0 r + Y_{\dot{v}_r} \dot{v}_r + Y_{v_r} v_r + Y_{\dot{r}} \dot{r} + Y_r r + Y_\delta \delta_r(t) \quad (16)$$

$$I_{zz}\dot{r} = N_{\dot{v}_r}\dot{v}_r + N_{v_r}v_r + N_{\dot{r}}\dot{r} + N_{r}r + N_{\delta}\delta_r(t) \quad (17)$$

When the equations are cast into matrix form of  $M\dot{x} = Ax + Bu$  the result is

$$\begin{bmatrix} m - Y_{\dot{v}_r} & -Y_{\dot{r}} \\ -N_{\dot{v}_r} & I_{zz} - N_{\dot{r}} \end{bmatrix} \begin{bmatrix} \dot{v}_r \\ \dot{r} \end{bmatrix} = \begin{bmatrix} Y_{v_r} & Y_r - mU_o \\ N_{v_r} & N_r \end{bmatrix} \begin{bmatrix} v_r \\ r \end{bmatrix} + \begin{bmatrix} Y_{\delta} \\ N_{\delta} \end{bmatrix} \delta_r(t) \quad (18)$$

In Equation 18 the term  $\delta_r(t)$  is a generalized command that represents the control input to both rudders. It should be noted that while the rudders act together they turn in opposite directions. This tandem but opposite operation allows for rapid turning during operation.

The final approximation made for the AUV was that the cross coupling terms in the mass matrix are zero. This approximation is based on the fact that the vehicle is symmetric and the rudders are very close to being equidistant from the body center. Therefore the final set of equations to be solved for steering behaviors are:

$$\begin{bmatrix} m - Y_{\dot{v}_r} & 0 \\ 0 & I_{zz} - N_{\dot{r}} \end{bmatrix} \begin{bmatrix} \dot{v}_r \\ \dot{r} \end{bmatrix} = \begin{bmatrix} Y_{v_r} & Y_r - mU_o \\ N_{v_r} & N_r \end{bmatrix} \begin{bmatrix} v_r \\ r \end{bmatrix} + \begin{bmatrix} Y_{\delta} \\ N_{\delta} \end{bmatrix} \delta_r(t) \quad (19)$$

The model is complete if values for the hydrodynamic coefficients can be found, and is appropriate for low angle of sideslip maneuvering in a longitudinal plane.

## B. AUV GEOMETRY, INERTIA PARAMETERS

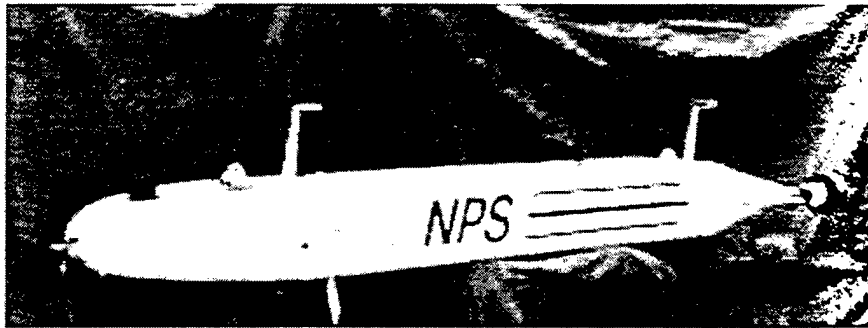
The usefulness of any model simulation is directly linked to the accuracy of the model being used. In theory an exact model will give an exact prediction of results for a particular simulation. However, in reality a perfect model, except in limited cases can never be achieved. The purpose of this section is to describe the NPS ARIES AUV, how

it was modeled and discusses assumptions and approximations that were made to develop the final model used.

### **1. AUV Model General Parameters**

The AIRES AUV shown in Figure 1 is the vehicle being modeled in this work. The parameters required for analysis include weight, volume, moment of inertia and hydrodynamic added mass. To begin modeling the AUV the overall dimensions were measured. Measured dimensions obtained were as follows:

1. Length from nose to end of tail section (not including thrusters) 128 inches.
2. Width determined longitudinally at the center section 16 inches.
3. Height determined vertically at center section 10 inches.
4. Weight was recorded on a lifting scale to be 490 lbs.



**Figure 1** ARIES AUV

The vehicle was then divided into three distinct sections for analysis. The tail section was modeled as a triangular prism. The center section was modeled as a rectangular prism. And finally, the nose section modeled as a half ellipsoid.

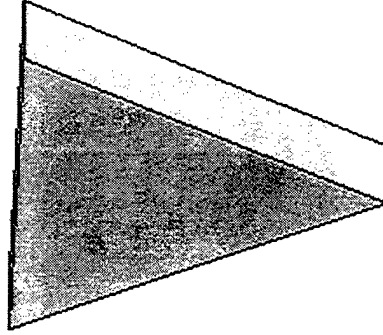
The assumption of symmetry for the AUV defines the body axes as the principal axis set for the vehicle. When evaluating the inertia matrix it then becomes diagonal ( $I_{xx}$ ,  $I_{yy}$ ,  $I_{zz}$ ).

## 2. AUV Tail Section Analysis

The AIRES AUV tail section was modeled as a rectangular prism shown in Figure 1. The dimensions used were based on where the vehicle started to change shape at the stern. For this and all subsequent discussions of parameters the x-axis is defined as the longitudinal or lengthwise direction (length nose to tail). The y-axis is defined to be the athwartships or beam direction (width). Lastly, the z-axis represents the overall height of the vehicle in the vertical direction (height). The values measured for calculations involving the tail section are:

- |           |           |
|-----------|-----------|
| a. Height | 10 inches |
| b. Width  | 16 inches |
| c. Length | 26 inches |

The calculation of volume was conducted by multiplying the area of the side face ( $\frac{1}{2} \text{Length} * \text{Height}$ ) by the width. Hence the total volume of the tail section was determined to be 2080 in<sup>3</sup>.



**Figure 2** Tail Section Model

The next step was to calculate the mass moment of inertia for the tail section. This required two steps. The first was to determine the moment of inertia for the section itself and the second was to translate the calculated moment to a selected reference point on the AUV. All moments of inertia were translated to the geometric center of the mid section. The moment of inertia for the tail section was not tabulated in Beer and Johnson (1997), Rothbart (1964) or any of numerous texts reviewed. The calculation was carried out using principals contained in Beer and Johnson (1997), these can also be found in most calculus textbooks, i.e. Finney and Thomas (1994). The solution for the moment of inertia in the x-direction was accomplished as follows:

First a differential mass ( $dm$ ) was chosen using a strip in the x-y plane. The origin was chosen to be on the forward face center. The differential equation of mass

$$dm = \rho c \left( -\frac{bx}{a} + b \right) dx \quad (20)$$

was developed based on geometry. Performing the integration of both sides yields the result.

$$m = \rho \frac{1}{2} abc \text{ or } m = \rho V \quad (21)$$

where  $m$  is the mass,  $\rho$  is the density,  $a$ ,  $b$  and  $c$  are the dimensions, and  $V$  is the volume.

For the elemental rectangular area in the  $y$ - $z$  plane,  $I_{z\text{area}} = \frac{1}{12} b^2 dm$ . Thus the differential mass moment of inertia is  $\rho t I_{z\text{area}}$  where  $t$  is the thickness of the differential mass.

Then the total differential mass moment of inertia ( $dI_z$ ) is,

$$dI_z = \rho t I_{z\text{area}} + x^2 dm. \quad (22)$$

Where the  $x^2 dm$  term translates the moment to the center of the AUV mid section using the parallel axis theorem as outlined in Beer and Johnson (1997). Integrating  $dI_z$  from  $x = 0$  to 26 inches for  $\rho t I_{z\text{area}}$  and  $x = 37.5$  to 63.5 inches for the  $x^2 dm$  term and substituting  $M$  for  $\rho V$ ,

$$I_{zz} = 2190.25 * M.$$

Similar calculations were performed for  $I_{xx}$  and  $I_{yy}$  using MAPLE software and are included in Appendix A. The resulting values are shown in Table 1.



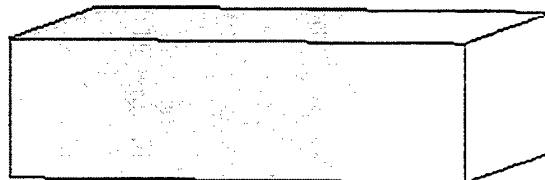
Table 1 Mass Moment of Inertia for AUV Tail Section

Axis	Mass Moment of Inertia	Units
$I_{xx}$	$142.33 \cdot M$	$\text{lb} \cdot \text{in} \cdot \text{sec}^2$
$I_{yy}$	$2177.25 \cdot M$	$\text{lb} \cdot \text{in} \cdot \text{sec}^2$
$I_{zz}$	$2190.25 \cdot M$	$\text{lb} \cdot \text{in} \cdot \text{sec}^2$

### 3. AUV Mid Section Analysis

The center section was treated as rectangular prism shown in Figure 3 whose dimensions are:

- a. Height            10 inches
- b. Width            16 inches
- c. Length           75 inches



**Figure 3** Mid Section Model

The formulas describing the mass moment of inertia for a rectangular prism as shown in Equations 23 thru 25 are readily available in many textbooks.

$$I_{xx} = \frac{1}{12} M(b^2 + c^2) \quad (23)$$

$$I_{yy} = \frac{1}{12} M(a^2 + c^2) \quad (24)$$

$$I_{zz} = \frac{1}{12} M(a^2 + b^2) \quad (25)$$

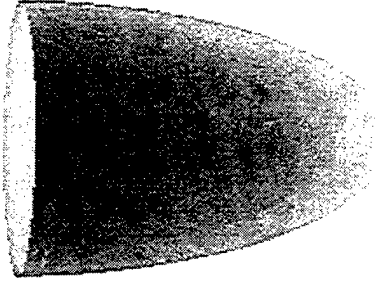
including Beer and Johnson (1997) and Finney and Thomas (1994). The derivation of the formulas is omitted and only the results are shown in Table 2 below.

Table 2 Mass Moment of Inertia for Mid Section

Axis	Mass Moment of Inertia	Units
$I_{xx}$	$29.67 \cdot M$	$\text{lb} \cdot \text{in} \cdot \text{sec}^2$
$I_{yy}$	$490.08 \cdot M$	$\text{lb} \cdot \text{in} \cdot \text{sec}^2$
$I_{zz}$	$477.08 \cdot M$	$\text{lb} \cdot \text{in} \cdot \text{sec}^2$

#### 4. AUV Nose Section Analysis

The moment of inertia calculation was carried out using the same methodology as the tail section. The model used for the nose shown in Figure 4 is that of a half ellipsoid.



**Figure 4** AUV Nose Section Model

First a differential mass ( $dm$ ) was chosen using a strip in the  $y$ - $z$  plane. The origin was chosen to be on the aft face center. The differential equation of mass

$$dm = \rho \pi b c \left( 1 - \frac{x^2}{a^2} \right) \quad (26)$$

was developed based on geometry of an ellipsoid. Performing the integration of both sides yields the result

$$m = \rho \pi a b c \text{ or } m = \rho V$$

where  $m$  is the mass,  $\rho$  is the density,  $a$ ,  $b$  and  $c$  are the radial dimensions, and  $V$  is the volume. For an elemental elliptical area in the  $y$ - $z$  plane,  $I_z \text{ area} = \frac{\pi}{4} z y^3$ , Beer and Johnson (1997). Thus the differential mass moment of inertia

$$dI_{zz, mass} = \rho t I_z \text{ area} = \rho (dx) \left( \frac{\pi}{4} z y^3 \right) \quad (27)$$

where  $dx$  is the thickness of the differential mass. Then the total differential mass moment of inertia ( $dI_z$ ) is,

$$dI_{zz} = \rho t I_z \text{ area} + x^2 dm . \quad (28)$$

Where the  $x^2 dm$  term translates the moment of the nose to the center of the AUV mid section using the parallel axis theorem as outlined in Beer and Johnson (1997). Integrating  $dI_{zz}$  from  $x = 0$  to 27 inches for  $\rho t I_z \text{ area}$  and  $x = 37.5$  to 63.5 inches for the  $x^2 dm$  term and substituting  $M$  for  $\rho V$ ,

$$I_{zz} = 2316.43 * M$$

Similar calculations were preformed for  $I_{xx}$  and  $I_{yy}$  using MAPLE software and are included in Appendix A. The resulting values are shown in Table 3 below.

Table 3 Mass Moment of Inertia for AUV Nose Section

Axis	Mass Moment of Inertia	Units
$I_{xx}$	$158.60 * M$	$\text{lb} * \text{in} * \text{sec}^2$
$I_{yy}$	$1789.85 * M$	$\text{lb} * \text{in} * \text{sec}^2$
$I_{zz}$	$2316.43 * M$	$\text{lb} * \text{in} * \text{sec}^2$

The value of density chosen for the AUV was based on its weight and displacement. To determine the displacement, a calculation was preformed to determine the volume of seawater for a weight equivalent to the AUV minus five pounds for buoyancy. Since the nose section had a free flood area the actual displacement of this section had to be estimated. To adjust the volume of the nose section a percent fill factor

was used to adjust the volume of the nose from a full volume to a displaced volume. Using a fill factor of forty percent resulted in a difference between calculated and estimated values of approximately one percent. A uniform density was then calculated by dividing the AUV weight by the calculated displaced volume. All calculations are listed in the *MATLAB* file *auvcalc.m* in Appendix B.

Table 4 shows the total mass moments of inertia as a result of summing the value of all three sections.

Table 4 Total Mass Moment of Inertia for the ARIES AUV

Axis	Mass Moment of Inertia	Units
$I_{xx}$	$330.60 \cdot M$	$\text{lb} \cdot \text{in} \cdot \text{sec}^2$
$I_{yy}$	$4457.18 \cdot M$	$\text{lb} \cdot \text{in} \cdot \text{sec}^2$
$I_{zz}$	$4983.76 \cdot M$	$\text{lb} \cdot \text{in} \cdot \text{sec}^2$

### C. AUV ADDED MASS PARAMETERS

This section will discuss the method used to obtain the added mass coefficients  $N_{\dot{r}}$  and  $Y_{\dot{v}}$  of Equation (19). All of the actual calculations are contained in the *MATLAB* file *auvcalc.m* in Appendix B. Added mass is a concept that represents the inertial reaction on a moving body in a dense medium arising from the time rate of change of fluid momentum. While many textbooks, such as White, 1999 have tables for some general shapes they do not have tables for combinations of these shapes or complex shapes. Due to the complexity of the shape of the AUV, an approximation of the overall shape had to be made. Initially, an oblate ellipsoid was chosen since the ability to specify

all three axis dimensions was desired. The initial reference used was Lamb, 1945. While the general description of how to solve this type of problem was addressed, no closed form solution existed. A report by Tuckerman, 1926 discussed the inertia factors of ellipsoidal shaped airships. This report discusses the complexity of the calculations and the use of elliptic integrals to obtain a solution. It was then decided to make a more general simplification to the model. The ARIES AUV was modeled as a prolate ellipsoid. This simplification allowed the use of tables and closed form solutions found in Lamb, 1945 to calculate the added mass terms needed.

The first step in calculating the added mass coefficients was to determine the neutrally buoyant mass. Total shape volume was calculated from the measured dimensions. The total weight was then equated to a volume of seawater. The purpose of this was to determine the difference of volumes since the nose is a free flood area. Results showed that a fill factor of forty percent was required to equate the two volumes. Since added mass calculations are based on displaced volume, the total volume was then equated to the prolate ellipsoid model. Requiring the volume to remain the same for the model, it was decided to maintain the overall length and let the equivalent diameter be calculated from the total volume. The calculated diameter along with the length was used in the eccentricity calculations. Formulas used also required the neutrally buoyant mass. The AUV was measured to have approximately five pounds of buoyancy. Therefore, five pounds was added to the measured weight to make the AUV neutrally buoyant for calculations.

The last step in calculating the added mass terms needed was to determine  $I_{zz}$ . The calculation of  $I_{zz}$  was used in the formulas developed in the previous sections but with

minor modifications. Since the AUV has lateral and vertical thruster tubes (that were empty when data was obtained) and no mass moment of inertia, the effect of these tubes was subtracted based on axis of rotation for modeling purposes. Table 5 lists the mass moment of inertia values calculated using the *MATLAB* file *auvcalc.m* listed in Appendix B.

Table 5 Final Mass Moments of Inertia for the ARIES AUV

Section	$I_{xx}$	$I_{yy}$	$I_{zz}$	Units
Nose	6.74	76.07	98.45	$\text{lb}\cdot\text{sec}^2\cdot\text{in}^{-1}$
Mid	33.44	552.25	537.59	$\text{lb}\cdot\text{sec}^2\cdot\text{in}^{-1}$
Tail	27.81	425.44	427.93	$\text{lb}\cdot\text{sec}^2\cdot\text{in}^{-1}$
Total (Eng)	67.99	1053.80	1064.00	$\text{lb}\cdot\text{sec}^2\cdot\text{in}^{-1}$
Total (SI)	7.68	119.10	120.26	$\text{kg}\cdot\text{m}^2$

As a check of values calculated for the mass moment of inertia, the radii of gyration was calculated for all three axes and found to lie within the vehicle body at reasonable points. The calculations are shown in the *MATLAB* file *auvcalc.m* listed in Appendix B and the results are listed in Table 6 below.

Table 6 Radii of Gyration for The ARIES AUV

	X Axis	Y Axis	Z Axis	Units
Distance (Eng)	7.32	28.81	28.95	in
Distance (SI)	18.60	73.20	73.56	cm

### III. PARAMETER IDENTIFICATION FROM VEHICLE MEASUREMENTS

#### A. INTRODUCTION AND BACKGROUND

The uncertainty of forces affecting any underwater vehicle motion requires knowledge of the methodology of parameter identification. Basically, the problem is to identify the desired vehicle hydrodynamic parameters from a system model and measurements of other vehicle parameters. This topic is discussed in control technology Soderstrum and Ljung (1983), Kalman (1969), Gleb (1974) and has been developed especially for dealing with this problem.

Essential to the parameter identification is an accurate model of the AUV control (input) and response (output). By manipulation of the equations of motion, we can let the system parameters become unknowns and the variables, as measured, to be known.

#### B. GENERAL THEORY

To model the AUV steering control, a model was selected with the input output response of the form

$$y(t) = H(t)\theta(t) + v(t), \quad (29)$$

where  $y(t)$  is the primary output,  $H(t)$  is a matrix of measurements relating the input to output, and  $\theta(t)$  being a parameter vector which is ideally constant. The term  $v(t)$  is a zero mean gaussian white noise.

Selecting a first order system as an example, and temporarily neglecting noise (avoiding differentiation problems) the model is:

$$\dot{x}(t) = \lambda(u(t) - x(t)) \text{ with } y(t) = cx(t).$$



Substituting and rewriting, the model becomes:

$$\dot{y}(t) + \lambda y(t) = c\lambda u(t), \text{ or in matrix form}$$

$$\dot{y}(t) = [-y(t) \quad u(t)]^* \begin{bmatrix} \lambda \\ \beta \end{bmatrix}, \quad H(t) = [-y(t) \quad u(t)] \text{ and } \theta(t) = \begin{bmatrix} \lambda \\ \beta \end{bmatrix}$$

where  $\dot{y}(t)$  is the derivative of the measured out put,  $\beta = c\lambda$  and the independent parameters are  $\beta$  and  $\lambda$ .

The AUV measurement data obtained from sensors is sampled at eight Hertz and is therefore not continuous. Since the data is sampled at a discrete interval, a discrete time model must be used. The discrete time transfer function of the above model is:

$$G(z) = \frac{Y(z)}{U(z)} = \frac{\sum_{i=1}^m b_i z^{-i}}{1 - \sum_{j=1}^n a_j z^{-j}}$$

and in time difference form the equation becomes:

$$y_t - a_1 y_{t-1} - a_2 y_{t-2} - \dots = b_1 u_{t-1} + b_2 u_{t-2} + \dots \quad \text{or} \quad y_t = [y_{t-1}, y_{t-2}, \dots, u_{t-1}, u_{t-2}, \dots] \quad (30)$$

so that  $H(t) = [y_{t-1}, y_{t-2}, \dots, u_{t-1}, u_{t-2}, \dots]$  and  $\theta(t) = [a_1, a_2, \dots, b_1, b_2, \dots]^T$ . The first equation represents a matrix containing the system inputs and outputs and the second is again a parameter vector.

### C. SOLUTIONS USING THE METHOD OF LEAST SQUARES

The purpose of using a least squares method is to estimate the parameters  $\theta(t)$ , which are never exactly known, so that the difference between the actual parameter and its estimate are minimized. This difference is called equation error and is given by,

$$e(t) = (y(t) - H(t)\hat{\theta}(t)) \quad (31)$$

where  $\hat{\theta}(t)$  is the estimate of  $\theta(t)$ . For the AUV this involves using the data obtained from the vehicles sensors combined with the system model shown in Equation (19) to obtain the hydrodynamic parameters. To minimize the error, define the scalar positive squared error measure,  $J(n) = \sum_{t=1}^n e'(t)e(t)$  then the minimization of  $J$  is given by

$$\frac{dJ}{d\hat{\theta}} = 0 = -\sum_{t=1}^n H'(t)e(t)$$

and substituting from Equation (29) yields

$$0 = -\sum_{t=1}^n H'(t)(y(t) - H(t)\hat{\theta}(t)) \quad (32)$$

Rearranging Equation (30) to get the generalized equation

$$\sum_{t=1}^n H'(t)y(t) = \sum_{t=1}^n H'(t)H(t)\hat{\theta}(t) \quad (33)$$

Writing the equations in matrix form and solving for  $\hat{\theta}(t)$  gives

$$\hat{\theta} = [H'H]^{-1}H'y \quad (34)$$

Using the matrix divide function in MATLAB to evaluate Equation (32) will give a result that is a least squares fit for  $\hat{\theta}(t)$ .

The same result may be found using gaussian random assumptions in which the solution,  $\hat{\theta}$  in Equation 33 is the most likely solution in which its probability is maximum Gelb (1974). It should be noted that the regression matrix,  $[\sum_{t=1}^n H'(t)H(t)]$  must be positive and strong (with no singularity) otherwise its inverse does not exist. This means that the system must be perpetually excited by its input.

## **IV. DATA ACQUISITION AND PROCESSING**

This chapter discusses the data collected by the ARIES AUV including the sensors. How the data is processed and the procedure used to obtain the hydrodynamic coefficients.

### **A. AUV DATA ACQUISITION**

An onboard computer samples all the data obtained from the AUVs sensors at eight hertz. This data is maintained onboard until the AUV surfaces and transmits the data through an aerial link, or it is downloaded by use of an interface cable connected to the AUV after recovery. The parameters logged by the AUV are post processed using the MATLAB script file in Appendix 2. All the parameters logged are listed below by sensor and variable name assigned to it. Some key variables recorded at 8 Hz. are

#### **1. Pressure Cell**

- a. Depth

#### **2. Gyro/Compass/Acoustic Doppler Log**

- a. Heading
- b. Forward Velocity (Relative to ground as well as the water mass)
- c. Sideslip Velocity (Relative to ground as well as the water mass)
- d. Heave Velocity (Relative to ground as well as the water mass)
- e. Roll Angle
- f. Roll Rate
- g. Pitch Angle
- h. Pitch Rate
- i. Yaw Angle (Heading) from Magnetic Compass
- j. Yaw Rate
- k. Altitude above bottom

#### **3. Global Positioning Receiver (GPS)**

- a. Position
- b. Heading

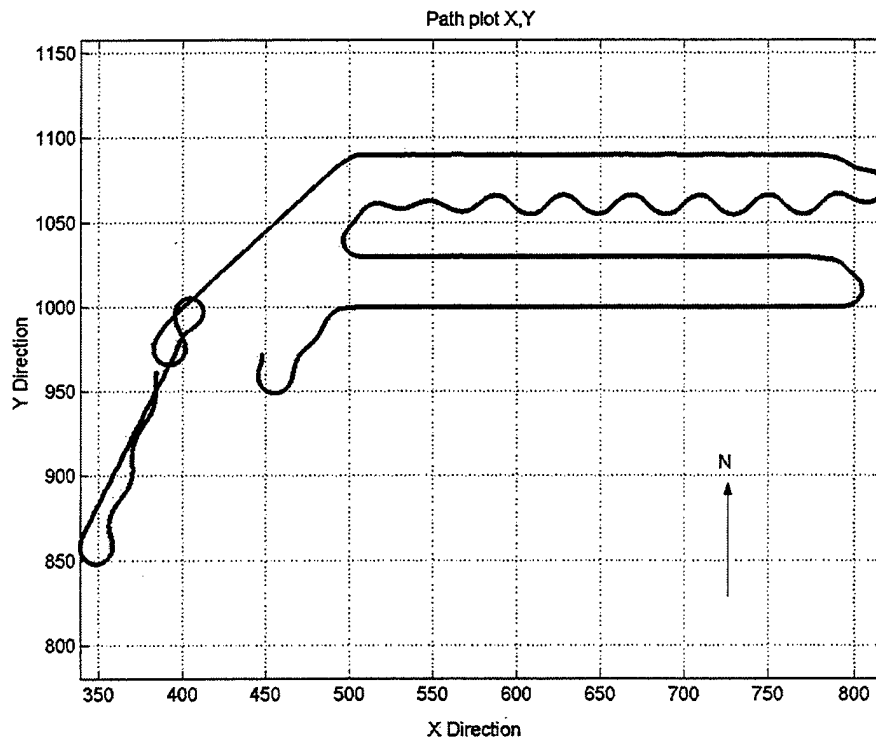
- c. Speed
- d. Differential Correction Status (0/1)

#### **4. Onboard Computer (Outputs from Controllers)**

- a. Rudder Position Ordered
- b. Plane Position Ordered
- c. Thruster Command Voltage

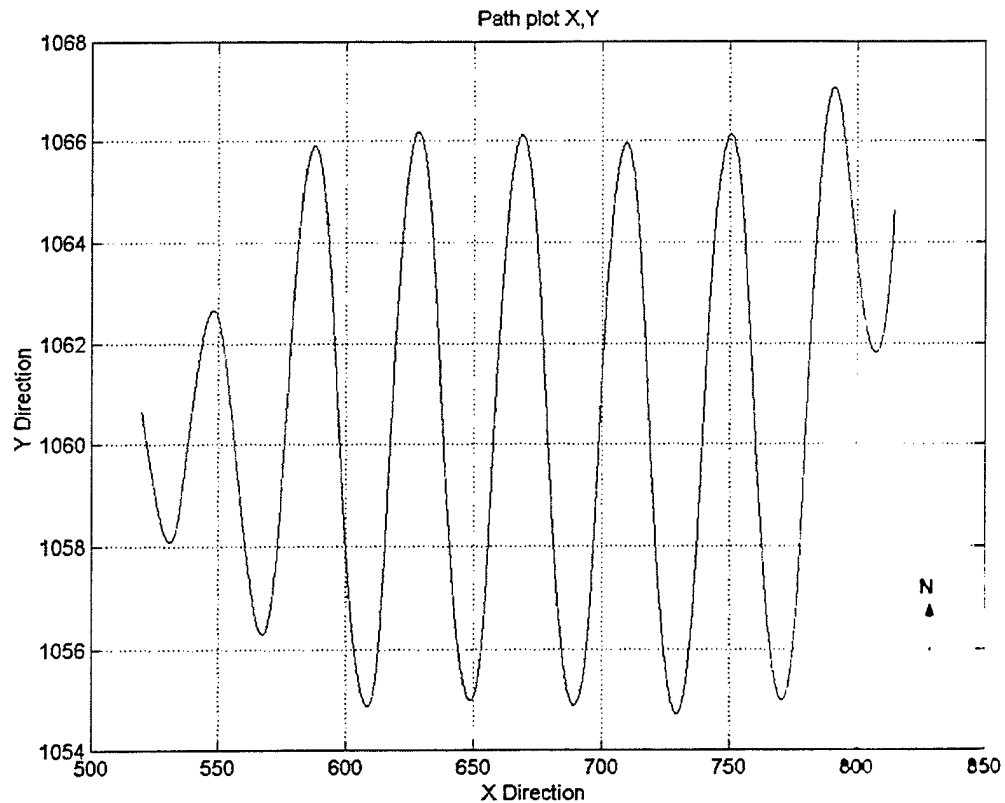
### **B. DATA EXTRACTION AND PROCESSING**

The raw data is post processed using MATLAB file assign38.m shown in Appendix B. The assign38.m file also produces a plot of vehicle position during the mission. Figure 5 shows the track of the ARIES AUV mission used in this report. For the analysis procedure used in this report, a set of continuously changing (dynamic) data was required. This data set represents a mission to track a set of waypoints forming a lawnmower pattern. On leg 3 an oscillating response produced data to identify the AUV parameters.



**Figure 5** *MATLAB* Plot of Full AUV Mission File d100600\_04.d

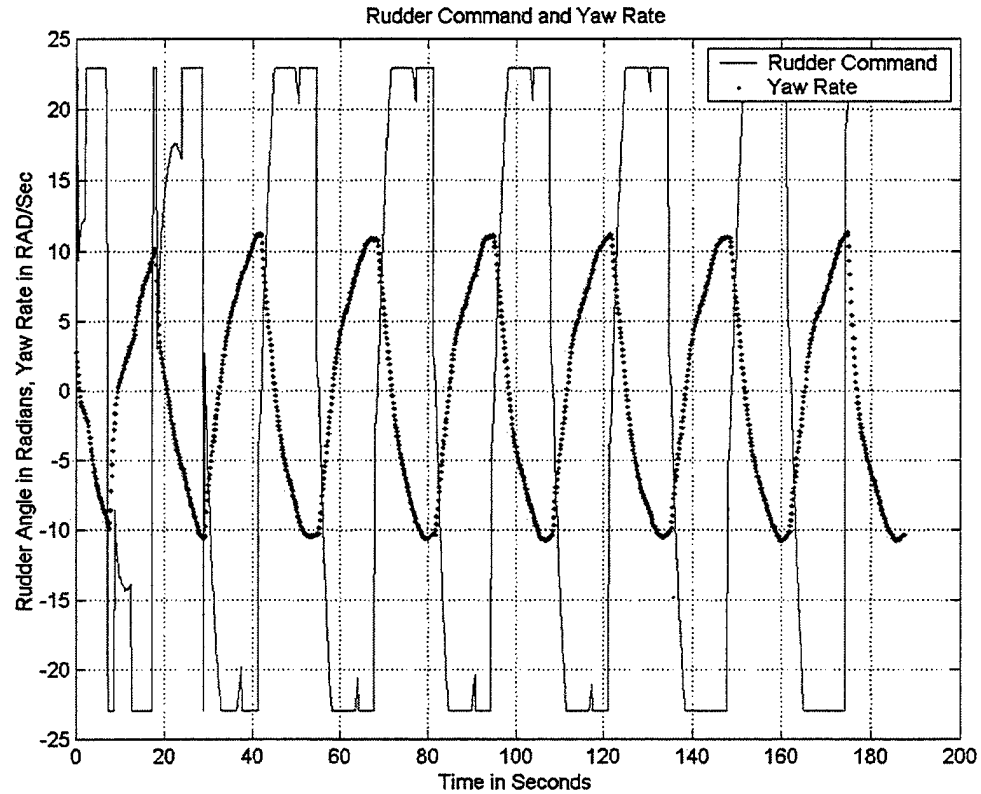
Figure 6 shows the subset of data seen in Figure 5 that is used in the determination hydrodynamic parameters. The limit cycling behavior arose from the use of an initial steering controller-not well behaved-but good for the purpose of steering parameter identification. Once this portion of the data was isolated for analysis it was analyzed for its usability.



**Figure 6** MATLAB Plot of AUV Mission File Portion Used In Analysis

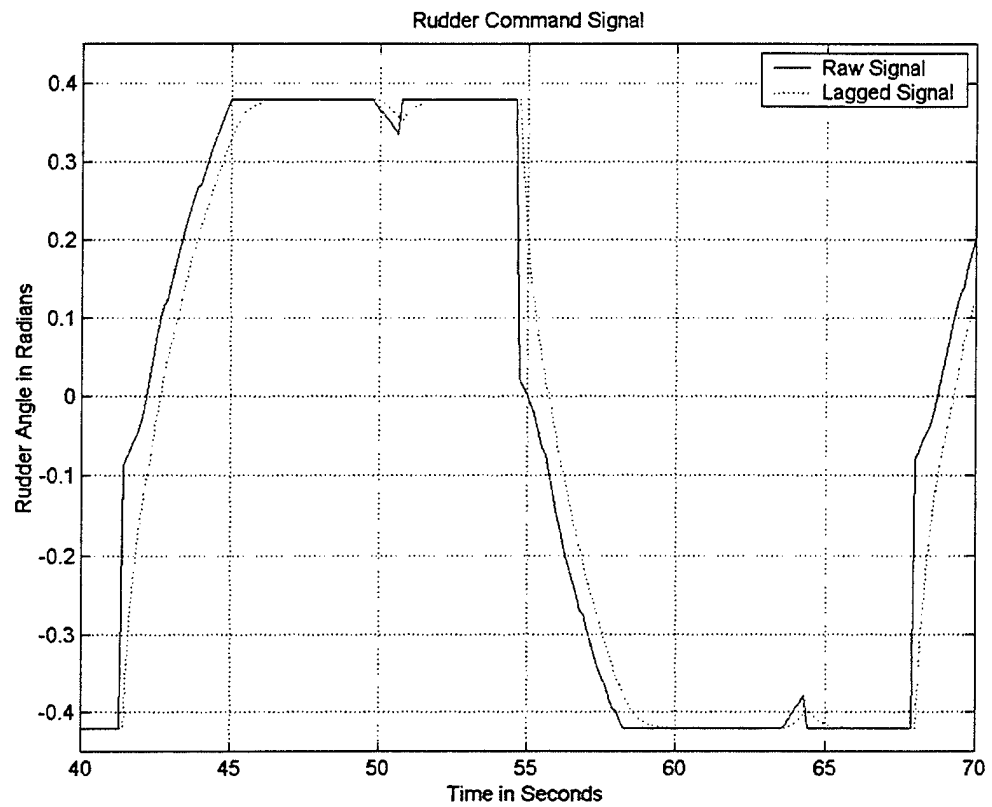
One of the limitations of the data was that the rudder signal was the command signal and not the actual position. The ARIES rudder motors are stepper motors run open loop. Figure 7 shows the recorded rudder command and yaw rate for the portion of the data being used. Looking closely at Figure 7, it can be seen that the change in yaw rate in response to rudder action was delayed. This delay is primarily the result of rudder response time. From the data shown in Figure 7 and observed response time, it was decided to use a first order lag of 0.26 seconds for the rudder signal. Figure 8 shows one rudder cycle and a first order lag version of the command signal. A first order lag was used to more accurately describe the time history required for the rudder to change position in response to the command signal. To reduce bias in the data, all parameters

were converted to zero mean before being used for calculations. The data is now ready to be used to calculate the hydrodynamic parameters. The *MATLAB* file *thesis\_values.m* in Appendix B was used to process the isolated data using mass matrix values calculated from Chapter II and Equation (19) developed in Chapter III.



**Figure 7** *MATLAB* Plot of AUV Rudder Order Command and Recorded Yaw Rate Used For Analysis





**Figure 8** *MATLAB* Plot of One Cycle Of AUV Rudder Order Command Original Data And Lagged Version Used For Analysis

## V. RESULTS

### A. DISCUSSION OF RESULTS

The first step in calculating was to put all of the variables into a set of consistent units. For this portion of the report all calculated and measured values were either verified or converted as required.

Then using Equation 19 and the assumption that there is no cross coupling and therefore the off diagonal terms of the mass matrix are zero, two independent differential equations can be written as follows:

$$(m - Y_{\dot{v}_r}) \dot{v}_r = Y_{v_r} v_r + (Y_r - m U_o) \dot{r} + Y_{\delta} \delta_r \quad (35)$$

$$(I_{zz} - N_{\dot{r}}) \dot{r} = N_{v_r} v_r + N_r r + N_{\delta} \delta_r \quad (36)$$

where the notation as a function of time has been omitted for clarity. Rewriting Equation 36 explicitly in terms of time and using  $IN = (I_{zz} - N_{\dot{r}})$ ,  $\alpha = N_{v_r}$ ,  $\beta = N_r$ , and  $\gamma = N_{\delta}$  results in

$$\frac{IN(r_{t+1} - r_t)}{\Delta t} = \alpha v_t + \beta r_t + \gamma \delta_t \quad (37)$$

Solving Equation 37 for  $r_{t+1}$  yields the result

$$r_{t+1} = \frac{\alpha \Delta t}{IN} v_t + \left( \frac{\beta \Delta t}{IN} + 1 \right) r_t + \frac{\gamma \Delta t}{IN} \delta_t \quad (38)$$

Now using the theory developed in Chapter III and relating Equation 38 to Equation 29 and 31 then,

$$\hat{\theta}(t) = [\theta_1(t), \theta_2(t), \theta_3(t)] , \quad y(t) = r_{t+1}, \text{ and } H(t) = [v_t, r_t, \delta_t]$$

where  $\theta_1 = \frac{\alpha \Delta t}{IN}$ ,  $\theta_2 = \frac{\beta \Delta t}{IN} + 1$  and  $\theta_3 = \frac{\gamma \Delta t}{IN}$ . Casting these results in the form Equation 33,

$\hat{\theta} = \left[ \frac{H' y}{H' H} \right]$  where  $H$  is a matrix and  $y$  is the vector  $r_{t+1}$ . Using *MATLAB* to perform the

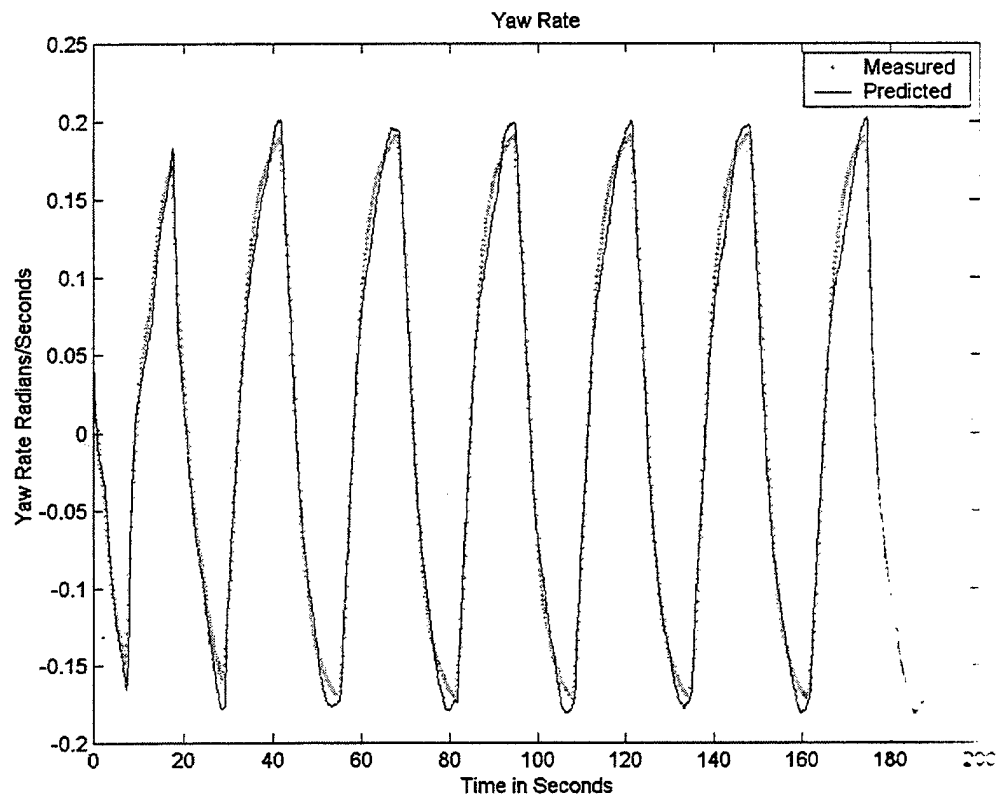
matrix operations gives a least squares fit for  $\hat{\theta}$  as discussed in Chapter III. From the values calculated for  $\theta_1$ ,  $\theta_2$  and  $\theta_3$ , the values of  $\alpha$ ,  $\beta$  and  $\gamma$  ( $N_{v_r}$ ,  $N_r$  and  $N_\delta$ ) are readily determined. Similar analysis was done for Equation 34 to obtain the sway parameters.

The *MATLAB* file *thesis\_values.m* calculated the hydrodynamic coefficients  $Y_{v_r}$ ,  $Y_r$ ,  $Y_\delta$ ,  $N_\delta$ ,  $N_{v_r}$  and  $N_r$ . The values obtained for these coefficients are summarized in Table 7.

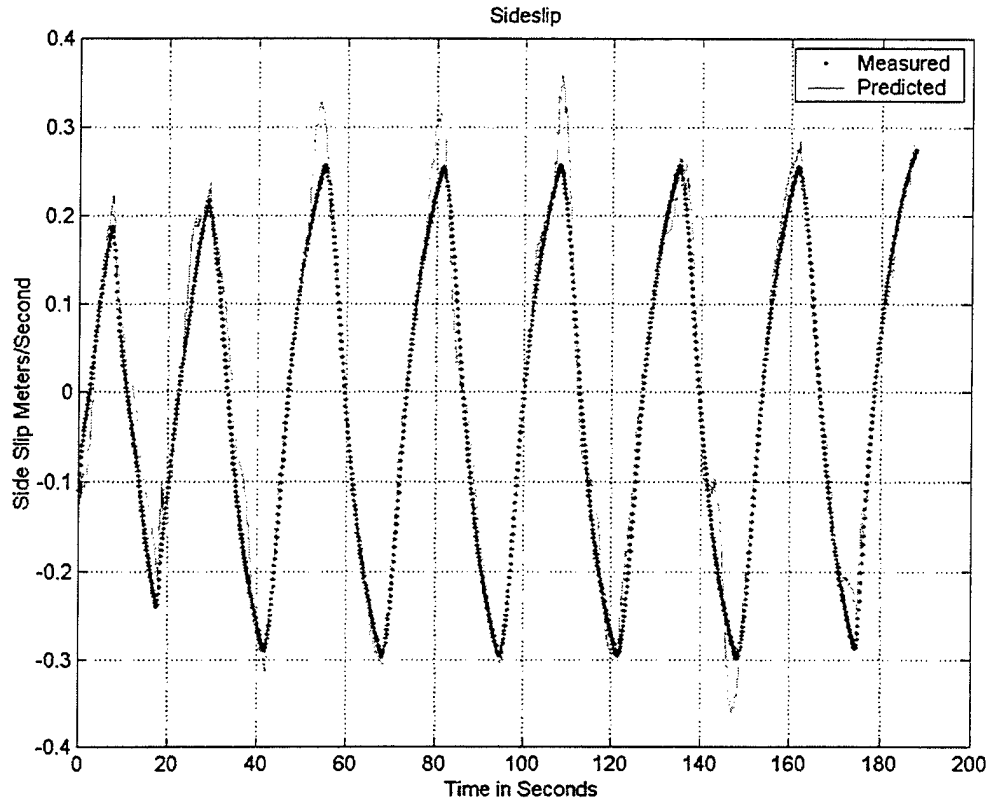
Table 7 Calculated Hydrodynamic Steering Coefficients of the ARIES AUV

$Y_{v_r}$	$Y_r$	$Y_\delta$	$N_{v_r}$	$N_r$	$N_\delta$
68.16	406.30	69.9	-10.89	-88.34	-35.47

Once these coefficients had been calculated they needed to be evaluated for accuracy. The coefficients and model developed were combined and a simulation was conducted to compare with the data obtained. Figure 9 shows the original yaw rate plot with the plot predicted by the model. Figure 10 shows the original sway plot with the plot predicted by the model. It can be seen that in both plots the two are very close to being the same, thereby showing the calculated coefficients are accurate.



**Figure 9** *MATLAB* Plot of Measured And Predicted Yaw Rate



**Figure 10** *MATLAB* Plot of Measured And Predicted Side Slip

## B. ERROR MEASURES

The scalar positive squared error measure  $J(n)$  (discussed in Chapter III C) for the coefficients obtained was  $6.2055 \times 10^{-5}$  for sway and  $5.3467 \times 10^{-6}$  for yaw rate. While this is a small number, the question remains, how does this compare to the data in a magnitude sense. To determine the relative error, the mean square error

$$J(n) = \frac{1}{N} \sum_{t=1}^n e'(t)e(t)$$

was divided by the square of the data's standard deviation  $\sigma_y^2$ . Taking the square root of this to get the relative error,

$$\text{rel} = \frac{\sqrt{\frac{1}{N} \sum_{t=1}^n e'(t)e(t)}}{\sigma_y} \times 100\%$$

results in a relative error of 4.38 and 1.79 percent in sway and yaw rate respectively.

Thus, the maximum error overall was less than 5 percent.

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## VI. CONCLUSIONS

The overall problem of underwater vehicle modeling is very complex. To obtain an accurate hydrodynamic model for any vehicle that is not a standard shape by purely analytical means is extremely difficult at best. The use of other methods such as use of model testing in tow tanks requires additional time, travel and expense and do not guarantee accurate results. Thus, the method used in this report has many positive features. The utility of this approach is to improve autopilot control design by using engineering approximations to the model of a vehicle. The data collected by the AUV can then be used to determine realistic estimates of the hydrodynamic parameters. An increased knowledge of hydrodynamic parameters then leads to a better control system design and performance.

While this approach requires that an initial steering controller be designed successfully so that the input-output data may be acquired, the results of this study has led to the design of an improved controller for which the limit cycling shown here is not present.



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## **APPENDIX A. *MAPLE* FILES FOR ARIES MASS MOMENT OF INERTIA CALCULATIONS**

This appendix contains *MAPLE* files for the calculation of the mass moment of inertias for the ARIES AUV nose, mid and tail sections respectively.

% Worksheet to calculate AUV nose section mass moment of inertia.

Calculate Ixx

```

> restart;                               Nose section equations
> L:=37.5+Y:                             Length from center of midsection to tip of nose section
> A:=27:                                 Length along x-axis
> B:=8:                                 Length along y-axis
> C:=5:                                 Length along z-axis
> dm:=pi*rho*A*C*(1-Y^2/B^2):           Differential mass
> x:=A*(1-Y^2/B^2)^0.5:                 Equation of ellipsoid
> z:=C*(1-Y^2/B^2)^0.5:
> m:=int(dm,Y=0..B):                   Calculate mass of section
> IAA:=pi/4*z*x^3*dy:                   IAA' for area slice
> dIAAm:=rho*IAA:                       Thickness = dy
> dIx:=dIAAm/dy+Y^2*dm:                 Calculate Ix
> dIxa:=dIx*M/m:                        M = m so answer is in form M*Ixx
> Ix:=int(dIxa,Y=0..B):
> simplify(Ix);                          Ixx
                                         158.6000000 M

```

Calculate Iyy

```

> restart;
> A:=27:
> B:=8:
> L:=37.5+Y:
> C:=5:
> dm:=pi*rho*A*C*(1-Y^2/B^2):           Differential mass
> x:=A*(1-Y^2/B^2)^0.5:                 Equation of ellipsoid
> z:=C*(1-Y^2/B^2)^0.5:
> m:=int(dm,Y=0..B):                   Mass
> IAA:=pi/4*z*x^3*dy:                   IAA' area for area slice
> dIAAm:=rho*IAA:                       Thickness = dy
> dIY:=dIAAm/dy+L^2*dm:
> dIYa:=dIY*M/m:
> IY:=int(dIYa,Y=0..B):
> simplify(IY);                          Iyy
                                         1789.850000 M

```

Calculate Izz

```
> restart;
> A:=27:
> L:=37.5+x:
> B:=5:
> C:=8:
> dm:=pi*rho*B*C*(1-x^2/A^2):      Differential mass
> y:=B*(1-x^2/A^2)^0.5:            Equation of ellipsoid
> z:=C*(1-x^2/A^2)^0.5:
> m:=int(dm,x=0..A):              Mass
> IAA:=pi/4*z*y^3*dx:              IAA' area
> dIAAm:=rho*IAA:                 Thickness = dx
> dIZ:=dIAAm/dx+L^2*dm:
> dIZa:=dIZ*M/m:
> int(dIZa,x=0..A);               Izz
                                   2316.425000 M
>
```

midsection.mws

%Worksheet to calculate AUV mid section mass moment of inertia.

> **restart;**

> **A:=75:** Length along x-axis

> **B:=16:** Length along y-axis

> **C:=10:** Length along z-axis

> **Ixx=1/12\*M\*(B^2+C^2) ;**

$$I_{xx} = \frac{89}{3} M$$

> **Iyy=1/12\*M\*(C^2+A^2) ;**

$$I_{yy} = \frac{5725}{12} M$$

> **Izz=1/12\*M\*(A^2+B^2) ;**

$$I_{zz} = \frac{5881}{12} M$$

>

tail.mws

% Worksheet used to calculate AUV tail section mass moments of inertia.

```

> restart;                                Tail Section Equations
>
>
>
> a:=26:                                Length along x-axis
> b:=16:                                Length along y-axis
> c:=10:                                Length along z-axis
> L:=x+37.5:                            Length from center of midsection to face
of tail section
> dm:=rho*c*(-b/a*x+b):                  Elemental mass
> m:=∫0a dm dx                          Calculate mass of section
> Iz_:= $\frac{1}{12} b^2 dm$                   Calculate Iz
> dIz:=Iz_+L2 dm
> Iz:=∫0a dIz dx
>
> simplify(Iz):
> MIz:= $\frac{Iz M}{m}$                         use M=m to get result in terms of
Mass*I
> simplify(MIz)                          Iz result
2190.250000 M

> Iy_:=1/12*c^2*dm:
> dIy:=Iy_+L2 dm                        Calculate Iy
> Iy:=∫0a dIy dx
> MIy:= $\frac{Iy M}{m}$ 
> simplify(MIy)                          Iy result
2177.250000 M

> Ix_:= $\frac{1}{12} (b^2 + c^2) dm$ 
> dIx:=Ix_+x2 dm                        Calculate Ix

```

$$> Ix := \int_0^a dIx \, dx$$

$$> MIx := \frac{Ix \, M}{m}$$

>

> simplify(MIx)

Ix result

$$\frac{427}{3} M$$

## **APPENDIX B. *MATLAB* FILES FOR DATA ANALYSIS**

This appendix contains the MATLAB script file for analyzing data acquired from the ARIES AUV. The first file assign38 converts data stored in the AUVs computer to a matrix form with variable handles. The second file auvcalc.m determines the mass matrix coefficients, converts units from English to SI, checks radius of gyration and incorporates modeling approximations. The file thesis\_values.m evaluates a predetermined portion of the data set and determines the hydrodynamic parameters based on values from auvcalc.m.



### assign38.m

```
clear all
clc

load d100600_04.d;

d=d100600_04;

State_Id = d(:,1);
Nav_Id   = d(:,2);
Month    = d(:,3);
Day      = d(:,4);
Year     = d(:,5);
Hour     = d(:,6);
Minute   = d(:,7);
Second   = d(:,8);
t        = d(:,9);
X        = d(:,10);
Y        = d(:,11);
Depth    = d(:,12);
Alt      = d(:,13);
phi      = d(:,14);
theta    = d(:,15);
psi      = d(:,16);
u        = d(:,17);
v        = d(:,18);
Depth_dot = d(:,19);
Alt_dot  = d(:,20);
p        = d(:,21);
q        = d(:,22);
r        = d(:,23);
```

```

Bias_psi = d(:,24);
Bias_r   = d(:,25);
Diff     = d(:,26);
NSv      = d(:,27);
LatDeg   = d(:,28);
LongDeg  = d(:,29);
TtTc    = d(:,30);
n_ls     = d(:,31);
n_rs     = d(:,32);
n_bvt    = d(:,33);
n_svt    = d(:,34);
n_blt    = d(:,35);
n_slt    = d(:,36);
delta_r  = d(:,37);
delta_sp = d(:,38);

```

```

dy=0.5;Y1=[Y(1):dy:491]; dx=(-19-X(1))/length((Y1)+1);

```

```

X1=[X(1):dx:X(1)+(length(Y1)-1)*dx];

```

```

Y2=[491:0.5:691];X2=-19*ones(length(Y2));

```

```

figure(1),clf,plot(Y,X,'b.', Y1,X1,'r.',Y2,X2,'r. '),grid, zoom, axis
equal;

```

```

title(' Path plot X,Y, blue, Desired Path in red');

```

```

figure(2), plot(t,delta_r,t,r,'r. '),grid, zoom

```

```

%analysis of model

rhat=r;

K=-0.1256;

lamda= 0.2363;

for i=1:(length(t)-1),

    rhat(i+1)=rhat(i)+0.125*(-lamda*rhat(i)+K*delta_r(i));

end;

figure(3),clf,plot(t,r,'b.',t,rhat,'r.',t,-delta_r./2,'k.'),grid

figure(4),clf, plot(t,atan2(v,u))

```

# auvcalc.m

```

%calculate volume of AUV
% all measurements calculated in inches
format compact
format short g
clc
total_wt=490/32.17/12;           % convert wt to mass
(lb s^2 in^-1)
sw_denisty=64/32.17/12^4;        %(lb s^2 in^-4)
Msw_denisty=1025;                %kg/m^3

batt_wt=23/32.17/12; batt_num=6; batt_vol=6*6*10;
% tail section
t_vert=10;
t_lat=16;
t_long=26;
t_vol=1/2*t_vert*t_lat*t_long    % 2080 in^3

% Thruster Vol (negative)
neg_vol=2*pi/4*10*5^2+2*pi/4*16*5^2; % 1021.02 in^3
neg_voly=2*pi/4*16*5^2;
neg_volz=2*pi/4*10*5^2;

% center section
c_vert=10; a1=27.5; a2=30.5; b1=17.5; b2=20.5;
c_lat=16;
c_long=75;

c_vol_total=c_vert*c_lat*c_long    %12000 in^3
c_vol=c_vert*c_lat*c_long-neg_vol  %10979 in^3
c_voly=c_vert*c_lat*c_long-neg_voly %11372 in^3
c_volz=c_vert*c_lat*c_long-neg_volz %11607 in^3

% nose section
fillfactor=0.4;
n_vert=10/2; n_lat=16/2; n_long=27/2;
n_vol=.5*4/3*pi*n_vert*n_lat*n_long*fillfactor % 452.4 in^3

% total
full_vol=c_vol_total+n_vol/fillfactor+t_vol    %15211
total_vol=t_vol+c_vol+n_vol                    % 13511 in^3
calc_vol=(total_wt+5/32.17/12)/sw_denisty      % 13365 5lb estimated
bouyancy
vol_error=(total_vol-calc_vol)/total_vol*100
homo_density=total_wt/total_vol                % 9.4e-5 lb s^2
in^-4
density_w_o_battery=(total_wt-batt_wt*batt_num)...
/(total_vol-batt_vol); %

%moment of inertia about center of mid section UNIFORM

c_Ix=89/3*homo_density*c_vol_total-...
1/12*homo_density*(3*2.5^2+10^2)-...
1/12*homo_density*(3*2.5^2+16^2) %33
c_Iy=5725/12*homo_density*c_vol_total...
```

```

- (1/12*homo_density*(3*2.5^2+10^2)+homo_density*b1^2)... %538
- (1/12*homo_density*(3*2.5^2+10^2)+homo_density*b2^2)...
- (1/2*homo_density*2.5^2+homo_density*a1^2)...
- (1/2*homo_density*2.5^2+homo_density*a2^2)
c_Iz=5881/12*homo_density*c_vol_total... %552
- (1/2*homo_density*2.5^2+homo_density*b1^2)...
- (1/2*homo_density*2.5^2+homo_density*b2^2)...
- (1/12*homo_density*(3*2.5^2+16^2)+homo_density*a1^2)...
- (1/12*homo_density*(3*2.5^2+16^2)+homo_density*a2^2)

%moment of inertia of nose about center of mid section UNIFORM
n_Ix=158.6*homo_density*n_vol %7
n_Iy=1789.85*homo_density*n_vol %76
n_Iz=2316.425*homo_density*n_vol %98

%moment of inertia of tail about center of mid section UNIFORM
t_Ix= 427/3*homo_density*t_vol %28
t_Iy= 2177.25*homo_density*t_vol %425
t_Iz= 2190*homo_density*t_vol %427

%total moment of inertia about center of mid section UNIFORM
Ix_total=t_Ix+c_Ix+n_Ix %68 1b s^2 in^-1
Iy_total=t_Iy+c_Iy+n_Iy %1039
Iz_total=t_Iz+c_Iz+n_Iz %1078

%check values of Ixx, Iyy & Izz

armx=sqrt(Ix_total/total_wt) %7.32 in
army=sqrt(Iy_total/total_wt) %28.62
armz=sqrt(Iz_total/total_wt) %29.15

%convert quantities to metric units

Mtotal_wt=490*0.4536 %222.26 kg wt of auv
MIxx=Ix_total*0.0254*.4536*9.81 %7.6849 kg m^2
MIyy=Iy_total*0.0254*.4536*9.81 %117.4
MIzz=Iz_total*0.0254*.4536*9.81 %121.9
Mvol_total=full_vol*.0254^3 %0.24926 m^3
ckx=sqrt(MIxx/Mtotal_wt)*100/2.54 %check of conversion 7.32
cky=sqrt(MIyy/Mtotal_wt)*100/2.54 %28.62
ckz=sqrt(MIzz/Mtotal_wt)*100/2.54 %29.16

%added mass calculations
L=64; W=8; H=5; %inches
%ecen=1-(((W+H)/2)/L)^2 %0.98 ave dia
ecen=1-((7.53/L)^2) %const vol const L 0.98616
alpha=2*(1-ecen^2)/ecen^3*(1/2*log((1+ecen)/(1-ecen))-ecen) %0.08583
beta=1/ecen^2-(1-ecen^2)/2/ecen^3*log((1+ecen)/(1-ecen)) %0.95709
kx=alpha/(2-alpha) %0.044639
ky=beta/(2-beta) %0.9177
kr=ecen^4*(beta-alpha)/((2-ecen^2)*(2*ecen^2-(2-ecen^2)*(beta-alpha)))
%0.76391
B_div_g=Mvol_total*Msw_denisty %kg mass of water vol moved
255.5
Xudot=-kx*B_div_g %kg -11.456
Yvdot=-ky*B_div_g %kg -234.47
Nrdot=-kr*MIzz %kg m^2 -93.13

```

Zwdot=Yvdot;	
Mqdot=Nrdot;	
Mmass=Mtotal_wt	%222.26 kg wt of auv
MY=Mmass-Yvdot	%456.73
IN=MIzz-Nrdot	%215.04

## thesis\_values.m

```

clear all
clc
format compact
load oscillation.mat;
%d=oscillation;

State_Id = d(:,1); Nav_Id = d(:,2); Month = d(:,3);
Day = d(:,4); Year = d(:,5); Hour = d(:,6);
Minute = d(:,7); Second = d(:,8); t = d(:,9);
X = d(:,10); Y = d(:,11); Depth = d(:,12);
Alt = d(:,13); phi = d(:,14); theta = d(:,15);
psi = d(:,16); u = d(:,17); v = d(:,18);
Depth_dot = d(:,19); Alt_dot = d(:,20); p = d(:,21);
q = d(:,22); r = d(:,23); Bias_psi = d(:,24);
Bias_r = d(:,25); Diff = d(:,26); NSv = d(:,27);
LatDeg = d(:,28); LongDeg = d(:,29); TtTc = d(:,30);
n_ls = d(:,31); n_rs = d(:,32); n_bvt = d(:,33);
n_svt = d(:,34); n_blt = d(:,35); n_slt = d(:,36);
delta_r = d(:,37); delta_sp = d(:,38);

%dy=0.5;Y1=[Y(1):dy:491]; dx=(-19-X(1))/length((Y1)+1);
%X1=[X(1):dx:X(1)+(length(Y1)-1)*dx];
%Y2=[491:0.5:691];X2=-19*ones(length(Y2));
%figure(1),clf,plot(Y,X)%, 'b.', Y1,X1,'r.',Y2,X2,'r. '),grid,zoom, axis
equal;
%title(' Path plot X,Y, blue, Desired Path in red');

% figure(2),plot(Y(2000:5000,1),X(2000:5000,1))
%figure(2), plot(t,delta_r,t,r,'r. '),grid,zoom

%analysis of model

%rhat=r;
K=-4;
lamda= 234;

%for i=1:(length(t)-1),
% rhat(i+1)=rhat(i)*lamda+K*delta_r(i);
%end;

%clf, plot(t,atan2(v,u))

MY=456.73
IN=215.04

rdr=delta_r*pi/180;
rdr=rdr-mean(rdr);
Rr=r*pi/180;
Rr=Rr-mean(Rr);
Vv=v-mean(v);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% lag rudder
y=[];

```

```

y(1)=rdr(1);
n=max(size(rdr));
p=0.26;
q=0.26;
for i=2:n
    y(i)=(1-p)*y(i-1)+q*rdr(i-1);
end
yrdr=y';
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%adjust v from nose to center
Lnose2center=55*2.54/100; %mid point length converted to meters
Mid_Vv=Vv-Lnose2center*Rr;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%cala theatas%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
H1=[Mid_Vv(1:1500,:),Rr(1:1500,:),yrdr(1:1500,:)];
Y1=[Mid_Vv(2:1501,:)];
theta1=inv(H1'*H1)*H1'*Y1

H2=[Rr(1:1500,:),yrdr(1:1500,:)];
Y2=[Rr(2:1501,:)];
theta2=inv(H2'*H2)*H2'*Y2
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
rr=[];rr(1)=Rr(1);
for i=1:1500, rr(i+1)=rr(i)+0.125*(-67.9/IN*rr(i)-32.4/IN*yrdr(i));end;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%calc paramenters
%
Yv=(theta1(1)-1)*8*MY %
Yr=theta1(2)*8*MY %
Yd=theta1(3)*8*MY %
%Nv=theta2(1)*IN*8 %
Nr=(theta2(1)-1)*IN*8 %
d=theta2(2)*IN*8 %

%check MX
det(H1'*H1) %4507
cond(H1'*H1) %422.36
%M=[IN];
%A=[Yr];
%MMM=M^-1*A
%eig(MMM) %[0.12268;6.9742]

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%check error
error1=Y1-H1*theta1;
J1=1/max(size(H1))*(error1'*error1) %6.22e-5

error2=Y2-H2*theta2;
J2=1/max(size(H2))*(error2'*error2) %5.41e-6

figure(2)
subplot(311)
plot(t,Rr,t,yrdr,':')
xlabel('v ... corrected for position')
subplot(312)
plot(H1(:,2))
xlabel('yaw rate')

```



```

subplot(313)
plot(error1,'r.')
xlabel('error')

%figure(3),plot(phi)
%clf,plot(t,r,'b.',t,rhat,'r.',t,-delta_r./2,'k.').grid
%figure(4), plot(Depth)

phidot=phi;
for i=2:1500
    phidot(i)=(phi(i+1)-phi(i-1))*4;
end

figure(6)
subplot(312)
plot(t,rr,'r.',t,Rr,'b')
subplot(311)
plot(error1)
subplot(313)
    plot(r,'r')

```

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